A Method of Stabilizing Cylinders in Fluid Flow

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The flutter motions which may occur when partly restrained circular cylinders are brought into uniform flow may be prevented by the installation of a splitter plate. The stabilizing principle of such a plate is explained and the optimum plate dimensions are discussed.

Nomenclature

A = projected area of cylinder
C_D = coefficient of drag
D = diameter of cylinder
f = frequency of vortex shedding
F_D = drag force
h = lateral vortex spacing in a Kármán vortex street
l = longitudinal vortex spacing in a Kármán vortex street
L = length of cylinder
L' = height of splitter plate
$L_s - 1$ = length of splitter plate in cylinder radii
P_n = stagnation point n (n = 1, 2, 3, 4, and 5)
R = Reynolds number
S = Strouhal number based on the diameter D and the velocity U
u = velocity component in x direction
U = approach velocity
v = velocity component in y direction
V = velocity of vortices relative to the fluid
w = complex potential
x_n = x coordinate of vortex n
$x_{\max}$ = x coordinate of $y_{\max}$ position
y_n = y coordinate of vortex n
$y_{\max}$ = maximum ordinate of the zero streamline
z = complex function $z(x,y) = x + iy$ where $i = (-1)^{1/2}$
$\bar{z}$ = conjugate of z
Γ = circulation of vortex
κ = strength of vortex
ρ = density of fluid
ρ_c = average density of cylinder
ρ_R = $\rho_R = (\rho + \rho_c)/\rho$
φ = velocity potential
ψ = stream function

Introduction

WHEN a bluff body is brought into the flow of a real fluid, vortex shedding will occur once a certain Reynolds number is exceeded. For a circular cylinder this Reynolds number is about 50. The frequency of shedding may be expressed as

$$f = SU/D \quad (1)$$

where the Strouhal number S is a function of the Reynolds number. Roshko[1] found

$$S = 0.212(1 - 21.2/R) \quad \text{for } 50 < R < 150 \quad (2)$$

and

$$S = 0.212(1 - 12.7/R) \quad \text{for } 300 < R < 2000 \quad (3)$$

while for the range of Reynolds numbers from 2000 to 400,000 the Strouhal number is reported to lie between 0.195 and 0.210.[2] A further increase in Reynolds numbers shows a transition zone,

$$4 \times 10^5 < R < 3.5 \times 10^6 \quad (4)$$

where the Strouhal number has been observed to lie between 0.21 and 0.46. For Reynolds numbers larger than 3.5×10^6 vortex shedding occurs with a Strouhal number of 0.27.[3]

The previously described alternate vortex shedding phenomenon gives rise to a fluctuating force acting on the cylinder in a transverse direction. The magnitude of this so-called Kármán force is

$$F_K = \tfrac{1}{2}\, C_K \rho A U^2 \quad (5)$$

where the coefficient C_K is usually assumed to be at least equal to 1.0.[4] Drescher[5] measured the unsteady pressure distribution of cylinders in crossflow, verifying the order of magnitude of the just-stated assumption.

The detrimental effect that such periodically alternating fluid flow may have or initiate on engineering structures became apparent by the collapse of a large suspension bridge, which was subjected to a moderate, steady wind. Other less dramatic and therefore less publicized failures are those of antennas, transmission lines, and smoke stacks. The recurrence of such failures is usually prevented by the installation of appropriate damping devices. In certain engineering applications, however, a change of the vibrational system, e.g., additional damping, is not sufficient and completely steady flow around the elastic or elastically supported structure or body may be desirable.

This report investigates a method to suppress the flutter motions that may occur when a circular cylinder is brought into uniform flow. The cylinder to be stabilized is slightly buoyant and its motion is only restricted by a cable of constant length. The cable is attached to the cylinder at the two ends of a pivot axle that runs through the center of drag of the cylinder (see Fig. 1). The cylinder will tend to keep its shown vertical position because the center of buoyancy lies above the center of gravity.

In the following it will be shown that the cylinder may be stabilized by means of a splitter plate (see Fig. 2). The minimum dimensions of such a fin will be derived and compared with the experimentally determined optimum dimen-

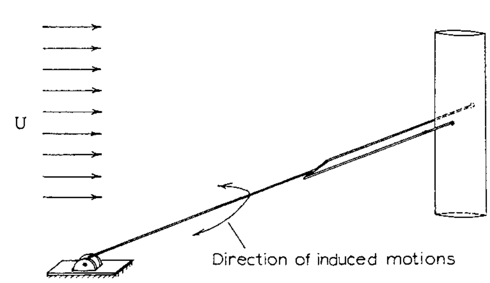

Fig. 1 Oscillating cylinder.

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sions. It should be realized, however, that a self-excited vibration may occur, e.g., on bodies with lift surfaces or with active mechanical systems, even though there is no periodic vortex shedding on the body which is initially steady. Preventing vortex shedding will eliminate the "switching mechanism"[6] most often responsible for the occurrence of the self-excited vibration of a body (other than airfoil-shaped) in steady fluid flow.

Welsh[7] showed in an extensive experimental investigation the stabilizing effect a splitter plate may have on the flutter motions of very short circular cylinders. His experiments were conducted with cylinders having a length to diameter ratio of 1.5. For cylinders of such small L/D ratios the end effects strongly influence the entire flow around the cylinder. Since the analysis of the splitter plate given below assumes two-dimensional flow, it is not valid for such short cylinders. Other methods of suppressing fluid induced oscillations by means of profile modifications of the cylinder are described by Grimminger[8] and by Price.[9] Grimminger found that two rigid plates attached to the cylinder in similar fashion as the splitter plate shown in Fig. 2, but each plate fixed 90° downstream from the front stagnation line, will reduce the flutter motions. Price[9] experimentally investigated a variety of profile modifications, such as attaching wires parallel to the cylinder, attaching short cylindrical fins to the cylinder as well as other modifications. In a preliminary experimental investigation this author found that most of the aforementioned methods (including the attaching of a fin having a length of 1 diam as was suggested for short cylinders) when tested on a long cylinder ($L/D = 5.15$) that was suspended as shown in Fig. 1, at best reduced but did not eliminate the vortex induced motions.

Stability Analysis

The underlying principle of the method by which a splitter plate of sufficient length and infinitesimal thickness will tend to inhibit vortex shedding becomes plausible from a stability analysis carried out by L. Föppl[10] in 1913.

Let the complex potential for two-dimensional flow be

$$w(z) = \varphi(x,y) + i\psi(x,y) \tag{6}$$

Then the complex velocity is

$$dw/dz = u - iv \tag{7}$$

where

$$u = \partial\varphi/\partial x = \partial\psi/\partial y \tag{8}$$

and

$$v = \partial\varphi/\partial y = -\partial\psi/\partial x \tag{9}$$

A closed circular streamline, with a radius equal to unity, representing the cylinder in uniform flow is formed by the complex potential

$$w_c = U(z + 1/z) \tag{10}$$

Adding to Eq. (10) the complex potentials of the vortices 1 and 2 located behind the cylinder and their respective images

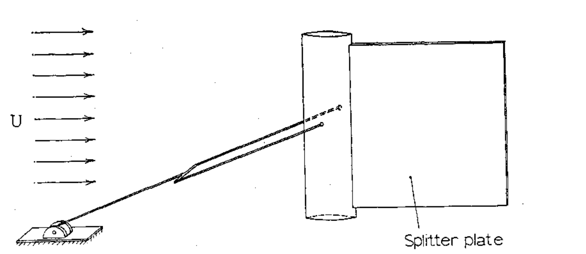

Fig. 2 Steady cylinder with stabilizing splitter plate.

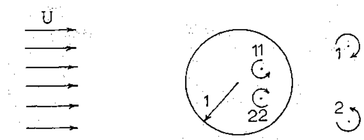

Fig. 3 Nomenclature of vortices.

11 and 22 (see Fig. 3) yields

$$w = U\left(z + \frac{1}{z}\right) + i\kappa \log \frac{(z - z_1)(z - z_{22})}{(z - z_2)(z - z_{11})} \tag{11}$$

It should be pointed out that by choosing a cylinder with a unit radius all distances are expressed in radii.

According to Thomson's theorem,

$$z_{11} = 1/\bar{z}_1 \tag{12}$$

and

$$z_{22} = 1/\bar{z}_2 \tag{13}$$

so that

$$x_{11,22} = x_{1,2}/(x_{1,2}^2 + y_{1,2}^2) \tag{14}$$

and

$$y_{11,22} = y_{1,2}/(x_{1,2}^2 + y_{1,2}^2) \tag{15}$$

By setting the propagation velocities of the vortices 1 and 2 equal to zero, Föppl[10] obtained

$$\pm 2y_{1,2} = r_{1,2} - 1/r_{1,2} \tag{16}$$

and

$$\kappa = 2Uy_{1,2}(1 - 1/r_{1,2}^4) \tag{17}$$

where

$$r_{1,2} = [x_{1,2}^2 + y_{1,2}^2]^{1/2} \tag{18}$$

and

$$r_1 = r_2 \tag{19}$$

since x_1 equals x_2 and y_1 equals $-y_2$ for the here assumed symmetrical case.

Equation (16) gives the loci at which the two vortices must be, if they are stationary with respect to the cylinder (see Fig. 4). Equation (17) correlates the strength κ of such a stationary vortex with its position behind the circular cylinder and the freestream velocity U. It is seen, that for a constant freestream velocity the vortex strength increases as the vortex pair moves away from the cylinder on the Föppl vortex path. Rubach's flow visualization experiments[11] agree with the theory developed by Föppl.

In order to determine whether Eq. (16) represents a stable equilibrium position of the vortices, vortices 1 and 2 are displaced by a small distance from their position on the Föppl path; a return to their original location means that this position is one of stable equilibrium. Such a stability analysis,

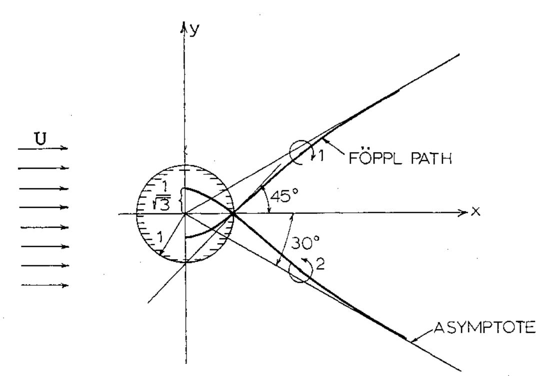

Fig. 4 Location of vortices.

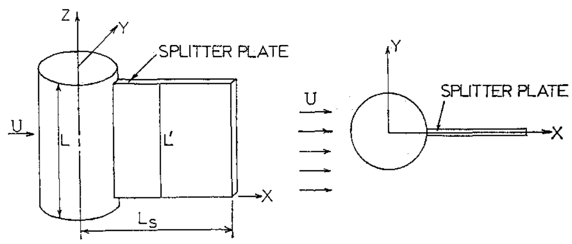

Fig. 5 Nomenclature of splitter plate.

which was carried out by Föppl,[10] showed that the vortex paths are positions of stable equilibrium as long as the vortex displacements are symmetrical with respect to the x axis. If the vortices are displaced asymmetrically with respect to the x axis, instability results, i.e., the vortices will not return to the Föppl path.

These considerations show that a pair of vortices may be at rest behind a circular cylinder in uniform flow. If this is the case, they will position themselves according to Eqs. (16) and (17), where the latter equation relates position with vortex strength and freestream velocity. When a disturbance acts upon these vortices, the vortices will swing back to their original position on the Föppl path, if the disturbance is of such a manner that it results in a small displacement of the vortices which is symmetrical with respect to the x axis. A disturbance producing an unsymmetrical displacement with respect to the x axis results in instability of the vortices. The typically staggered Kármán vortex street is usually formed in this case.

The reason why a splitter plate may prevent vortex shedding is now apparent. Because of the presence of the splitter plate, one vortex (vortex 1 or vortex 2) does not "see" the opposite vortex (vortex 2 or vortex 1, respectively). The splitter plate has an image effect, so that each vortex sees its own image instead of the real opposite vortex. Since the splitter plate extends along the x axis, the displacements of the image vortices and their respective real vortices are therefore always symmetrical to the x axis.

Splitter Plate Dimensions

The question now arises what the optimum dimensions of such a splitter plate should be, in order to prevent vortex shedding. The answer to this question is important to the practical application of this method of vortex shedding prevention (see Fig. 5).

The optimum dimensions are the shortest length and the shortest height of the splitter plate which will prevent vortex shedding in the real fluid in which the cylinder moves with uniform velocity. It is desirable, however, to deduce from theoretical considerations the minimum plate dimensions defined below. Such considerations will not only shorten the experimental investigation but will also yield valuable results of general character.

The minimum fin length ($L_s - 1$) which will insure the separation of the fluid moving with the vortices is given by the location of the rear stagnation point P_3 (see Fig. 6). This minimum fin length, which is defined for the nonviscous two-dimensional flowfield, is shorter than the optimum fin length which was defined for the real, viscous flow in consideration. The difference between the minimum fin length and the optimum fin length is not only caused by viscous effects; a finite displacement of vortex 1 or 2 in the downstream direction will require a corresponding increase in the minimum fin length to insure complete separation of the two fluid bodies moving with the circular zero streamline representing the cylinder. An expression for the distance L_s may now be derived (see also Ref. 11). Separating Eq. (11) into real and imaginary parts and comparing with Eq. (6) yields

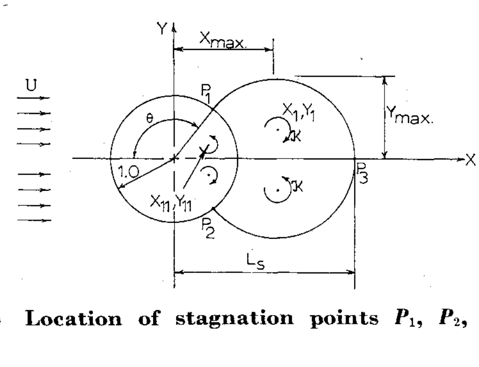

Fig. 6 Location of stagnation points P_1, P_2, and P_3.

$$\varphi = U\left(x + \frac{x}{x^2 + y^2}\right) + \kappa\left(\arctan\frac{y + y_1}{x - x_1} + \arctan\frac{y - y_{11}}{x - x_{11}} - \arctan\frac{y - y_1}{x - x_1} - \arctan\frac{y + y_{11}}{x - x_{11}}\right) \quad (20)$$

and

$$\psi = U\left(y - \frac{y}{x^2 + y^2}\right) + \kappa \log\left\{\frac{[(x - x_1)^2 + (y - y_1)^2][(x - x_{11})^2 + (y + y_{11})^2]}{[(x - x_1)^2 + (y + y_1)^2][(x - x_{11})^2 + (y - y_{11})^2]}\right\}^{1/2} \quad (21)$$

By performing the operations indicated in Eqs. (8) and (9) the velocities in the x and y directions are found to be

$$u = U\left(1 - \frac{x^2 - y^2}{(x^2 + y^2)^2}\right) + \kappa\left[\frac{y - y_1}{(x - x_1)^2 + (y - y_1)^2} + \frac{y + y_{11}}{(x - x_{11})^2 + (y + y_{11})^2} - \frac{y + y_1}{(x - x_1)^2 + (y + y_1)^2} - \frac{y - y_{11}}{(x - x_{11})^2 + (y - y_{11})^2}\right] \quad (22)$$

$$-v = \frac{2U\,x\,y}{(x^2 + y^2)^2} + \kappa\left[\frac{x - x_1}{(x - x_1)^2 + (y - y_1)^2} + \frac{x - x_{11}}{(x - x_{11})^2 + (y + y_{11})^2} - \frac{x - x_1}{(x - x_1)^2 + (y + y_1)^2} - \frac{x - x_{11}}{(x - x_{11})^2 + (y - y_{11})^2}\right] \quad (23)$$

Equating Eq. (22) to zero and setting $y = 0$ and $x = L_s$ gives the required expression containing the term L_s,

$$1 - \frac{1}{L_s^2} + \frac{2\kappa}{U}\left[\frac{y_{11}}{(L_s - x_{11})^2 + y_{11}^2} - \frac{y_1}{(L_s - x_1)^2 - y_1^2}\right] = 0 \quad (24)$$

To determine the value of L_s from Eq. (24), the location of the vortices and the ratio κ/U must be known. However, since a stationary vortex pair exists, only one vortex coordinate must be given or be assumed, as Eq. (16) determines the second coordinate and Eq. (17) then in turn determines the corresponding κ/U ratio. Conversely, if the ratio of vortex strength to freestream velocity is known, the position of the stationary vortices may then be calculated with Eqs. (16) and (17), and Eq. (24) may again be solved for L_s.

From this discussion, it is apparent that if the numerical value for L_s is to be obtained, either the vortex position or κ/U, the ratio of vortex strength to freestream velocity must be assumed. Naturally, a complete solution of the Navier-

Table 1 Flowfield parameters[a]

θ	x_{I}	y_{I}	x_{II}	y_{II}	κ/U	L_s	$x_{\max}$	$y_{\max}$	Assumption
95.0	11.514	6.597	0.0654	0.0375	13.194	22.94	11.513	13.756	
100.0	5.836	3.270	0.1304	0.0731	6.537	11.5	5.835	6.825	
105.0	3.968	2.144	0.1951	0.1054	4.278	7.68	3.965	4.474	
110.0	3.058	1.575	0.2584	0.1331	3.127	5.78	3.052	3.283	← 2 (laminar boundary layer—potential flow velocity)
115.0	2.520	1.221	0.3214	0.1558	2.403	4.62	2.510	2.544	
115.2	2.503	1.21	0.3239	0.1566	2.380	4.58	2.492	2.520	← 4 (laminar boundary layer)
120.0	2.169	0.980	0.3829	0.1730	1.899	3.85	2.153	2.036	
125.0	1.920	0.800	0.4438	0.1848	1.514	3.28	1.897	1.660	
129.5	1.750	0.67	0.4984	0.1908	1.231	2.88	1.719	1.389	← 4 (turbulent boundary layer)
130.0	1.738	0.661	0.5025	0.1911	1.212	2.85	1.708	1.370	← 2 (turbulent boundary layer)
133.7	1.634	0.578	0.5440	0.1924	1.028	2.60	1.597	1.20	← 3 (laminar boundary layer)
135.0	1.598	0.549	0.5596	0.1923	0.964	2.50	1.560	1.136	
140.0	1.482	0.452	0.6171	0.1883	0.748	2.22	1.436	0.937	
145.0	1.382	0.365	0.6761	0.1787	0.5560	1.97	1.329	0.756	
145.7	1.375	0.359	0.6807	0.1776	0.5421	1.95	1.321	0.743	← 1 (Kármán vortex)
150.0	1.3067	0.297	0.7277	0.1653	0.4096	1.77	1.250	0.658	
155.3	1.2370	0.232	0.7809	0.1465	0.2791	1.59	1.15	0.50	← 3 (turbulent boundary layer)
160.0	1.186	0.183	0.8236	0.1273	0.1898	1.46	0.952	0.360	

[a] All length dimensions are expressed in cylinder radii, and all angles are given in degrees.

Stokes equations for flow around a cylinder at a Reynolds number at which vortex shedding is about to start will yield the rear stagnation point P_3 directly, without neglecting the deformation of the vortices which occurs in real fluid flow. In this article, however, the following more or less plausible assumptions are considered.

1) The κ/U ratio of a stationary vortex is the same as that of a single vortex in a Kármán vortex street. This assumption will give a smaller than actual κ/U ratio, which is apparent from a simple energy consideration; therefore, a smaller than adequate length L_s will result.

2) The stagnation points P_1 and P_2 coincide with the positions of boundary-layer separation of a circular cylinder in uniform flow (see Fig. 6). In this case the splitter plate length will vary substantially, depending on whether a laminar or a turbulent boundary layer, giving early or delayed separation respectively, exists.

3) The distance $2y_{\max}$ (see Fig. 6) is the same as the width of the wake behind a long circular cylinder of equivalent diameter.

4) The vortices behind the cylinder have the same lateral spacing as the vortices in a staggered Kármán vortex street.

For the first of these assumptions, a value of κ/U for a single vortex of a Kármán vortex street must be deduced. The strength of a line vortex is

$$\kappa = \Gamma/2\pi \tag{25}$$

where

$$\Gamma = 2Vl \cdot \text{cotan}(\pi h/l) \tag{26}$$

Substituting the Kármán stability value[12]

$$h/l = 0.283 \tag{27}$$

into Eq. (26) gives

$$\Gamma = 2[2]^{1/2}lV \tag{28}$$

Applying the Krönauer stability criterion, the velocity V may be expressed in terms of the velocity U. For the vortex spacing ratio given in Eq. (27), a ratio of

$$V/U = 0.14 \tag{29}$$

is obtained,[13] which is the same value observed by von Kármán in his original experiments supporting his theory.[12] The vortex strength to freestream velocity may now be expressed as

$$\kappa/U = (0.14[2]^{1/2}/\pi)l \tag{30}$$

The longitudinal vortex spacing l in a Kármán vortex street is shown to be[14]

$$l = 0.86\, D/S \tag{31}$$

or

$$l = (D/2S)(1 + [1 - C_DS/0.397]^{1/2}) \tag{32}$$

depending on whether Eq. (29) is assumed to be correct or if one relies only upon empirically found values for the drag coefficient and the Strouhal number. The results obtained by Eqs. (31) and (32) are in close agreement with each other and with the experimental evidence.[14] For the Reynolds number range $2 \times 10^4 < R < 2 \times 10^5$ the generally accepted values for a long circular cylinder are $S = 0.195$ and $C_D = 1.20$, resulting in the numerical value for l of $4.31D$. Since the radius of the cylinder was defined to be unity, l equals 8.62 and the required ratio is

$$\kappa/U = 0.542 \tag{33}$$

Table I shows the resulting length L_s, the position of the vortices and stagnation points, the κ/U ratio and the coordinates of the maximum width of the zero streamline.

The second assumption to be investigated is based on the geometrical similarity of the potential and real fluid flowfields. Terminating the Blasius power series with the fifth term, separation ($du/dy = 0$) occurs at $\theta = 110°$, when potential flow velocity is assumed to exist.[15] In the present comparison, a less conventional definition of separation is more suitable. Fage[16] shows that the visually observed separation of the boundary layer may take place as much as 10° downstream from the angle θ determined by the point on the cylinder at which inflection of the velocity profile occurs ($du/dy = 0$). In real fluids separation takes place further upstream in the laminar flow regime and further downstream in the turbulent flow regime, due to the particular pressure distribution formed. For this and the aforementioned reasons a variety of values for θ have been assumed and the corresponding flowfield characteristics computed as shown in Table 1. Since geometrical similarity is emphasized, the visually observed separation point should be used when estimating the minimum fin length in this case.

The third assumption compares the wake width of the real fluid with the maximum width of the zero streamline of the potential flowfield. For the specific parameters the reader

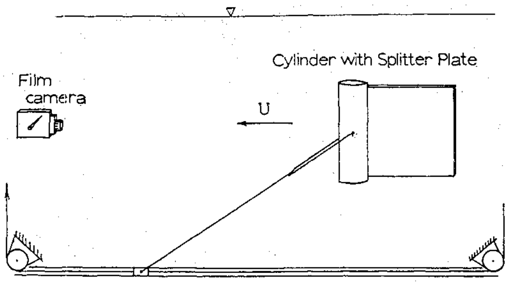

Fig. 7 Test arrangement showing cylinder with splitter plate.

is again referred to Table 1. Based on Roshko's result[17] that the wake Strouhal number for circular cylinders is 0.16, the wake width in a Reynolds number range of 2×10^4 to 2×10^5 is 1.2 diam. Above the critical Reynolds number the wake is found to be 0.5 diam wide.

The fourth assumption proposes the same lateral vortex spacing of the real, staggered Kármán vortex street and the two vortices behind the cylinder. Using the mean values for l as obtained from Eqs. (31) and (32) by substituting the coefficients

$$S = 0.195, \quad C_D = 1.20 \quad \text{for } 2 \times 10^4 < R < 2 \times 10^5$$

and

$$S = 0.370, \quad C_D = 0.35 \quad \text{for } R \approx 10^6$$

the y coordinates for the vortex positions on the Föppl path become $y_1 = 1.21$ and $y_1 = 0.67$ for the previously indicated cases, respectively.

The second dimension of the splitter plate L' may be estimated by relating the decrease in the coefficient of drag of a finite cylinder to the decrease in vortex length parallel to the axis of the cylinder,[14]

$$L' = \frac{C_D \text{ (finite } L/D)}{C_D \text{ (infinite } L/D)} L \qquad (34)$$

Discussion

Inspection of Table 1 shows that assumptions 2 and 4 result in similar flowfields not only when a laminar but also when a turbulent boundary layer exists. Assumptions 1 and 3 also result in flowfields having parameters of approximately the same magnitude; here is it noticed that the values obtained by assumption 1 lie between those obtained by assumption 3, which vary according to the type of existing boundary layer.

Experiments performed with water as a medium tend to support the theoretical results as obtained with assumptions 2 and 4. A cylinder with a L/D ratio of 5.15 and a density ratio ρ_R equal to 1.85 was towed at different speeds in a large tank and its motions filmed (see Fig. 7). For towing speeds for which the Reynolds number based on the cylinder diameter was below the critical, a splitter plate of length $3D$ corresponding to L_s equal to 7 was sufficient to eliminate all motions induced by the vortex shedding, while test runs with a fin of length $2D$ (equivalent to $L_s = 5$) showed the typical oscillating motions of a bare cylinder. Although the $3D$ fin stayed aligned with the direction of flow (all fins were rigidly attached to the cylinder), the $2D$ fin appeared to support an oscillation of the cylinder around its longitudinal axis. For velocities for which the Reynolds numbers were well above the critical, a splitter plate of $1.5D$ ($L_s = 4.0$) was sufficient to suppress all vortex induced motions. The aforementioned theoretical considerations tend to give a less than necessary splitter plate length rather than the exact or even a longer than necessary length. The reason for this was already mentioned.

It should be pointed out that the observation of the motion of the cylinder is not a direct observation of the vortex shedding and its prevention by a splitter plate, but rather an observation of the reaction of a complex, self-excitable vibrational system when vortex shedding has been inhibited to various degrees by different splitter plates. Experimental evidence based on flow visualization should therefore be obtained. However, the primary objective, namely stabilization of the cylinder in fluid flow, was achieved. Although for the bare cylinder flutter amplitudes of up to $\pm 3D$ were observed, the attachment of a splitter plate having the proper length reduced this amplitude to less than $\pm 0.01D$. Tow tests were also performed using different cable lengths, changing the natural frequency of the pendulumlike system (see Fig. 7). In this manner, it was assured that the natural frequency and the vortex shedding frequency (possibly reduced to a lower value due to the splitter plate according to Ref. 18) were of such magnitudes that the typical self-excited vibrations with large amplitudes would have occurred. Meier-Windhorst[19] shows that the frequency range for which the large amplitudes occur is strongly dependent upon the density ratio ρ_R, becoming wider as ρ_R approaches 2. The same important trend was also noticed by Glass[20] and was further investigated by Sallet,[21] again supporting the validity of a deduction of the flowfield in view of vortex shedding from the observations of the motion of the cylinder.

The previously described tests also indicated that the other minimum dimensions of the splitter plate is $L' = 0.9L$ for subcritical flow and $L' = L$ for velocities where the Reynolds number exceeds the critical, as is plausible from Eq. (34) in conjunction with the drag data of circular cylinders having the indicated slenderness ratio (e.g., Ref. 22).

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